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1982 J. Phys. A: Math. Gen. 15 1561

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J. Phys. A: Math. Gen. **15** (1982) 1561–1565. Printed in Great Britain

Quantum electrodynamic theory of voltage carrying states in a current biased Josephson weak link

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Received 22 June 1981, in final form 10 December 1981

Abstract. The Feynman path integral formulation of the Josephson effect is applied to the problem of the influence of quantum electrodynamic fluxoid tunnelling on the characteristics of a current biased, singly connected, Josephson weak link. The well known Josephson voltage biased frequency effect, $\omega_V = (2eV/\hbar)$, has a natural duality extension to a current bias frequency effect $\omega_I = (\pi I/e)$.

The phenomenon of fluxoid (or phase) quantisation in 'SQUID' (superconducting quantum interference device) rings is well established (Silver and Zimmerman 1967). In its simplest form the SQUID consists of a thick superconducting ring enclosing a Josephson weak link. If the weak link critical supercurrent is I_c, and the ring inductance is L, then the inequality $2\pi LI_c/c\Phi_0 > 1$, where $\Phi_0 = (\pi hc/e) = 2.07 \times 10^{-7}$ gauss cm^2, sets the condition that externally generated magnetic flux will enter the ring discontinuously, in units of Φ_0. At this point the diamagnetic screening supercurrent I_s in the ring equals I_c. In the usual classical theory of SQUID rings the external flux is allowed to enter the ring since when $I_s = I_c$, the weak link is considered to have been driven into the normal state. The flux (in units of Φ_0) then slides through this normal link in a completely classical manner accompanied by a corresponding reduction in I_s. Subsequently, the superconducting state is re-established in the link (Rieger *et al* 1972). Phenomenologically, the transit time for this flux transfer is taken to be L/R, where R is the so-called shunt resistor in the resistively shunted junction model of the weak link (Jackel and Buhrman 1975). This flux entry model provides the basis for the usual, classical description of AC-biased SQUID magnetometers.

Recently we reported (Prance *et al* 1981) experimental evidence that magnetic flux could enter a SQUID ring in which $2\pi LI_c/c\Phi_0 \ll 1$ at screening currents $I_s \ll I_c$. We attributed this to quantum electrodynamic tunnelling of magnetic flux bundles (again in units of Φ_0) across the Josephson weak link in the SQUID ring. In this quantum electrodynamic model of flux entry, tunnelling of the flux occurs via a virtual intermediate state in which momentarily there is pair charge separation at the weak link. During this separation the flux slides through quantum mechanically.

The experiments of Prance *et al*, although concerned with the particular problem of quantum electrodynamic tunnelling of Faraday magnetic flux bundles into a SQUID ring, do lend interest to generalised macroscopic quantum phase interference phenomena. Thus, the Feynman path integral view of the macroscopic phase, which was used to interpret these experiments, gives rise to the notion that Josephson critical

0305-4470/82/051561+05$02.00 © 1982 The Institute of Physics

transport laws appear in duality pairs. Quantum electrodynamic transport yields, as a dual pair, the original Josephson critical current law together with a 'critical voltage' law (Widom and Clark 1980, Widom *et al* 1981). Thus in this quantum electrodynamic model of the SQUID ring the photon wavefunction Hamiltonian for the ring has the form of a double canonical pendulum,

$$H_{\text{photon}} = (Q^2/2C) - \hbar\Omega \cos(\Phi_0 Q/\hbar c) + (\Phi - \Phi_x)^2/2L - \hbar\nu \cos(q\Phi/\hbar c), \tag{1}$$

where $\Omega(T)$ is the coherent part of the flux tunnelling matrix and $\nu(T)$ is the lowest-order harmonic Josephson pair tunnelling frequency. Here Φ and Φ_x are, respectively, the total included flux in the SQUID ring and the external flux applied to the ring. The (instantaneous) total included charge on the Josephson weak link capacitor is denoted by Q and the pair charge quantum $(2e)$ by q.

The above formulation incorporates both the well known 'critical current' Josephson law

$$I(T) = q\nu \sin(2\pi\Phi/\Phi_0), \tag{2}$$

for charge pairs passing through the SQUID ring weak link, and a 'critical voltage' law for the voltage across this weak link,

$$V_c(T) = (\Phi_0\Omega(T)/c)\sin(2\pi Q/q). \tag{3}$$

In this model, voltage noise $>\Phi_0\Omega(t)/c$ destroys coherent quantum electrodynamic flux tunnelling just as in the familiar Josephson relation (equation (2)) current noise $>q\nu(T)$ destroys coherent pair charge tunnelling. The relation between $\Omega(T)$ and $\nu(T)$ is of the form (Prance *et al* 1981)

$$\Omega(T) = \beta\pi^2\nu(T)\exp - (\Phi_0^2\langle\Delta Q^2\rangle/2\hbar^2) \tag{4}$$

where the charge fluctuation on the weak link capacitor, value C, is determined by an effective quantum noise temperature T^*, defined by the equipartition rule

$$\langle\Delta Q^2\rangle = Ck_B T^*. \tag{5}$$

It is obvious from equations (4) and (5) that the value of the quantum electrodynamic critical voltage depends crucially on the weak link capacitance and effective SQUID ring noise temperature. Thus, for a typical point contact SQUID ring, we estimate (Prance *et al* 1981) $V_c \sim 0.1\ \mu V$ at a thermal equilibrium (i.e. bath) temperature of 4.2 K.

In what follows, it will be shown that the Josephson oscillations at a bias frequency ω_b (the 'AC' effect) also appear in dual pairs, depending on whether a Josephson weak link is voltage or current biased. This latter duality takes on importance for experimentally detecting the coherent quantum electrodynamic tunnelling of Faraday flux bundles across a current biased weak link.

The concept of a bias frequency follows easily from the Feynman–Schwinger (Feynman *et al* 1965) action principle formulation of quantum mechanics stated as follows. The amplitude for a process P modified by an action ΔS is related to the amplitude for the same process without modification by

$$\text{Amp}(P, \Delta S) = \text{Amp}(P, 0)\exp(i\Delta S/\hbar). \tag{6}$$

A frequency bias is an action modification of the form

$$\Delta S = -t\Delta E, \tag{7}$$

i.e. the bias frequency is

$$\omega_b = (\Delta E/\hbar). \tag{8}$$

In computing transition rates per unit time, the bias frequency enters into the law of energy conservation as

$$W^{\pm}_{i\to f} = (2\pi/\hbar^2)\,|T_{fi}|^2\,\delta(\omega_b \mp \omega_{fi}), \tag{9a}$$

$$\hbar\omega_{fi} = E_f - E_i, \tag{9b}$$

where T_{fi} is the transition amplitude (in energy units) and the $(\pm)$ refers to a process and its time reversal counterpart.

The quantum electrodynamic duality for a frequency biased weak link (Josephson 1962, 1964, Likharev 1979) is as follows.

(i) For the tunnelling of electron pairs

$$q = 2e, \tag{10}$$

through a voltage biased weak link, the energy modification is given by

$$E(\text{pair}) = qV, \tag{11}$$

which leads, via equations (8), (10) and (11), to the Josephson bias frequency

$$\omega_V = (2eV/\hbar). \tag{12}$$

(ii) For the tunnelling of Faraday magnetic flux bundles of strength

$$\Phi_0 = (\pi\hbar c/e) \tag{13}$$

across a biased weak link, the energy modification is given by

$$\Delta E(\text{fluxoid}) = (\Phi_0 I/c), \tag{14}$$

which leads, via equations (8), (13) and (14) to the dual bias frequency

$$\omega_I = (\pi I/e). \tag{15}$$

The power dissipation in a system induced by a frequency bias is given by the detailed balance of gain and loss transitions, i.e. the heating rate is

$$\dot{Q} = \sum_i \sum_f p_i(\hbar\omega_{fi})(W^{+}_{i\to f} - W^{-}_{i\to f}) \tag{16}$$

which, by virtue of equations (9) and (16), can be expressed in terms of the spectral functions

$$\int_{-\infty}^{+\infty} S^{+}(\omega)\,e^{-i\omega t}\,d\omega = \langle T^{+}(t)T(0)\rangle, \tag{17a}$$

$$\int_{-\infty}^{+\infty} S^{-}(\omega)\,e^{-i\omega t}\,d\omega = \langle T(0)T^{+}(t)\rangle, \tag{17b}$$

as

$$\dot{Q} = (2\pi/\hbar)\omega_b[S^{+}(\omega_b) - S^{-}(\omega_b)]. \tag{18}$$

The operator amplitude commutator implied by equations (17) and (18) has the usual fluctuation-response theorem interpretation.

Let us return to the quantum electrodynamic duality, where the heating rate in equation (18) is a mean voltage–current product. In a voltage biased weak link the pair current admittance $Y_{\text{pair}}(\zeta)$ describes the pair tunnelling current fluctuations. At a frequency bias less than the amount required to break up pairs,

$$\hbar\omega_{\text{r}} < 2\Delta \tag{19a}$$

or

$$V < (\Delta/e), \tag{19b}$$

the pair current response is determined by the dissipation in the admittance as

$$I_{\text{pair}}(V) = V \operatorname{Re} Y_{\text{pair}}(\omega_{\text{v}} + \mathrm{i}0^{+}). \tag{20}$$

The quantum electrodynamic flux tunnelling dual to equation (20) is that the impedance $Z_{\text{flux}}(\zeta)$ describes fluxoid tunnelling voltage fluctuations. At a current source frequency bias less than the amount required to break up fluxoids, i.e. for $I < I_{\text{c}}$ or

$$\omega_{\text{I}} < (\pi I_{\text{c}}/e), \tag{21}$$

where I_{c} is the critical current for turning the weak link normal, the fluxoid voltage response is determined by dissipation in the impedance as

$$V_{\text{flux}}(I) = I \operatorname{Re} Z_{\text{flux}}(\omega_{\text{I}} + \mathrm{i}0^{+}). \tag{22}$$

For a system obeying the critical voltage law,

$$Z_{\text{flux}}(\zeta) = \frac{1}{C^*}\left(\frac{\tau}{1 - \mathrm{i}\zeta\tau}\right) \tag{23}$$

where C^* is the 'effective' capacitance, related to the flux tunnelling frequency Ω by

$$\Omega = (e^2/\hbar c)(c/\pi^2 C^*), \tag{24}$$

and $\tau = R^* C^*$ is the effective dissipative lifetime intrinsic to the flux tunnelling event. Equations (17) and (23) imply that

$$V_{\text{flux}}(I) = \frac{1}{C^*}\left(\frac{\tau I}{1 + (\omega_{\text{I}}\tau)^2}\right), \tag{25a}$$

$$I \ll I_{\text{c}}. \tag{25b}$$

We cannot, as yet, provide an estimate for τ, since although the experiments of Prance *et al* show that quantum electrodynamic flux tunnelling events are taking place, these do not provide information on the level of coherence involved.

Equations (15), (24) and (25) constitute a current–voltage characteristic anomaly, which is the signature for quantum electrodynamic tunnelling in a singly connected Josephson weak link. Such a signature, which is shown diagrammatically in figure 1, should be susceptible to experimental test. It should be noted, however, that even if our estimate of the critical voltage in weak links ($\simeq 0.1\ \mu$V for $C \simeq 3 \times 10^{-15}$ F, $T^* \simeq 2$ K and a critical current $\simeq 10\ \mu$A) is reasonable, the observation of the characteristic quantum electrodynamic current–voltage anomaly may still prove difficult. In our experience, even the observation of the true, as distinct from noise averaged, flux-dependent behaviour of SQUID rings requires a combination of massive electromagnetic shielding, battery-operated low noise power supplies and ultra low noise

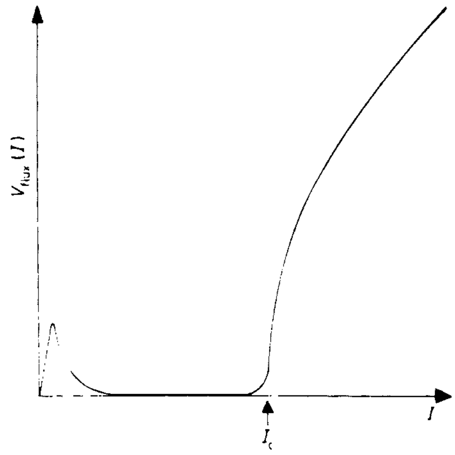

Figure 1. Current versus voltage curve, shown schematically, for a current biased Josephson weak link displaying quantum electrodynamic tunnelling. The predicted voltage bump should be observed for $I \ll I_c$. This figure is meant to illustrate the form of the critical voltage anomaly, not to set its size.

temperature (<10 K) liquid helium cooled preamplifiers (Prance *et al* 1982). SQUID rings, coupled to *in situ* helium cooled amplifiers, are several orders of magnitude easier to isolate from the effects of noise then singly connected weak links. To our knowledge, all observations of the low-frequency current–voltage characteristics of weak links have been performed with noisy room temperature commercial amplifiers. The usual practice in such experiments is to provide low-frequency (up to ~1 kHz) filtering of the voltage and current leads. However, it is impossible to remove the effects of mains noise unless the whole system (weak link, cryostat, sweep and record systems, DC power supplies, etc) are massively shielded magnetically. Furthermore, it is the total integrated noise up to many GHz which will determine whether or not 'critical voltage' phenomena can be seen. It is the usual practice to provide bandstop filters for room temperature (environmental) noise injected down the bias and voltage leads for a very limited range of frequencies (e.g. ~1 kHz). Only by very careful design of the total experimental system will integrated noise voltages <0.1 μV be attained.

References

Feynman R P *et al* 1965 *Lectures on Physics* vol III, ch 21 (Reading, Mass: Addison-Wesley)
Jackel L D and Buhrman R A 1975 *J. Low Temp. Phys.* **19** 201
Josephson B D 1962 *Phys. Lett.* **1** 251–2
—— 1964 *Rev. Mod. Phys.* **36** 216
Likharev K K 1979 *Rev. Mod. Phys.* **51** 101–59
Prance R J, Clark T D, Mutton J E, Prance H, Hughes J, Widom A and Megaloudis G 1982 *Macroscopic Spectroscopy of Free Energy Levels in a Superconducting Quantum Interference Device* in preparation
Prance R J, Long A P, Clark T D, Widom A, Mutton J E, Potts M W, Megaloudis G and Goodall F 1981 *Nature* **289** 543–9.
Rieger T J, Scalapino D J and Mercereau J E 1972 *Phys. Rev.* B 6 1734
Silver A H and Zimmerman J E 1967 *Phys. Rev.* **157** 317
Widom A and Clark T D 1980 *Lett. Nuovo Cimento* **28** 186
Widom A, Megaloudis G, Sacco J E and Clark T D 1981 *Nuovo Cimento* **61B** 112–21